Quantum technologies:
Building a Quantum Simulator

Mher Ghulinyan

Functional Materials & Photonic Systems

Luciano Serafini

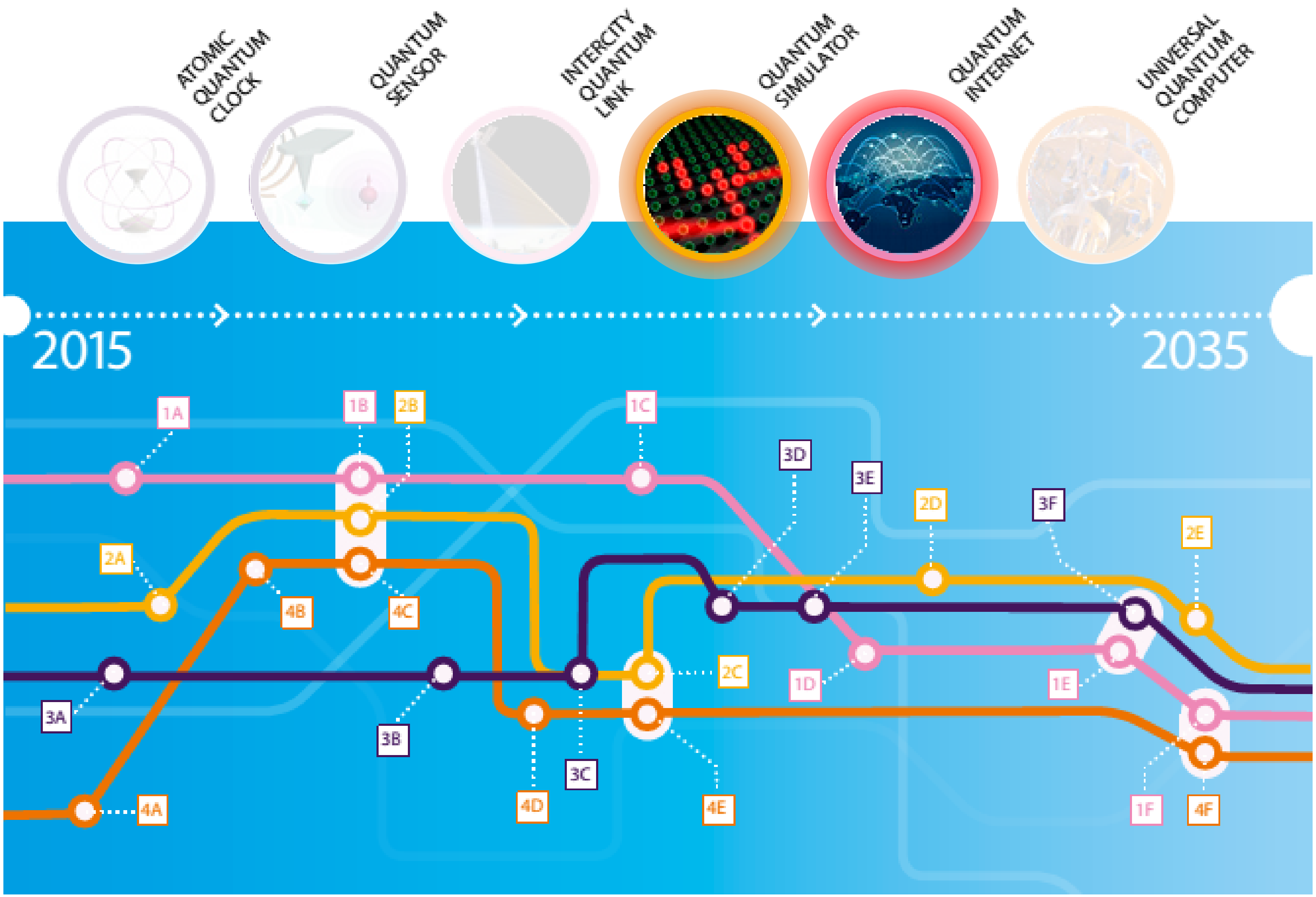
Data and Knowledge Management

http://qurope.eu/manifesto

This manifesto is a call to launch an ambitious European initiative in quantum technologies, needed to ensure Europe's leading role in a technological revolution now under way.

Europe needs strategic investment now in order to lead the second quantum revolution. Building upon its scientific excellence, Europe has the opportunity to create a competitive industry for long-term prosperity and security.

Quantum Technologies Timeline



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Outline

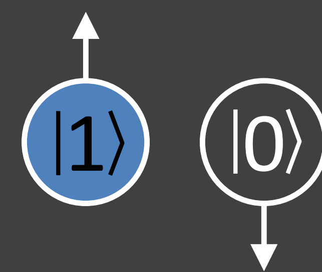
- ❑ Why “quantum”?
- ❑ Bit vs Qubit
- ❑ Logic gates
- ❑ optical C-NOT quantum gate
- ❑ Reconfigurability of a Q-circuit
- ❑ Quantum simulators
- ❑ the project INQUEST
  - Hardware – Quantum simulator
  - Software – Quantum algorithms

Why Quantum and not Classical?

Classical computation – data unit is bit

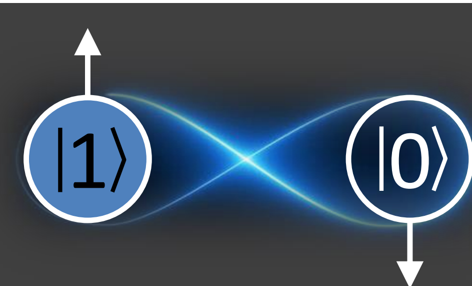
Valid output

Quantum computation – data unit is qubit



Valid output

$$|\psi\rangle = \alpha \times |0\rangle + \beta \times |1\rangle$$



Qubit – a two-state *quantum-mechanical* system

- Polarization of a single photon (↑ up or ↓ down)

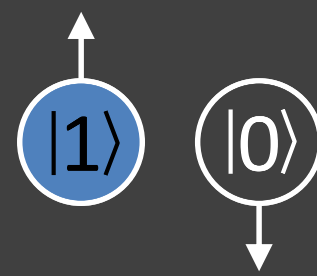
Superposition of two states:

Probability
$|0\rangle \rightarrow \alpha^2$; $|1\rangle \rightarrow \beta^2$

$$\alpha^2 + \beta^2 = 1$$

store much more information than just 1 or 0, because they can exist in any superposition of these values.

Quantum computation – data unit is qubit



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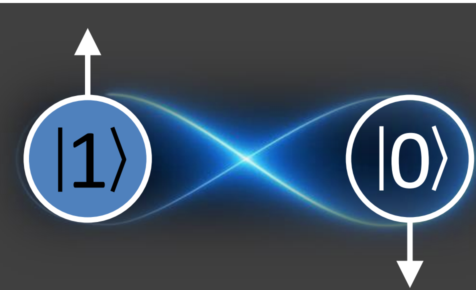

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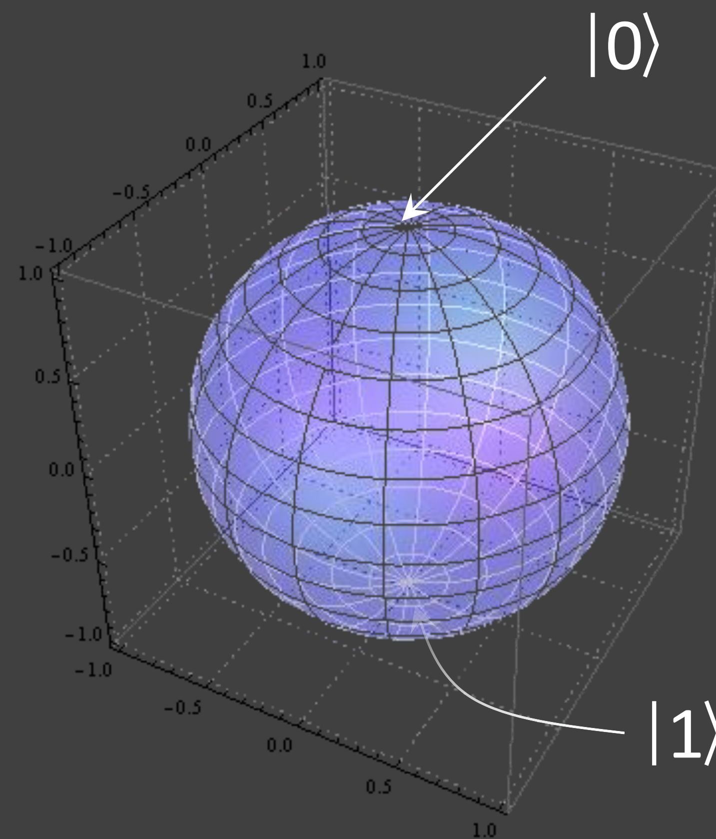
Why Quantum and not Classical?

Valid output

Valid output



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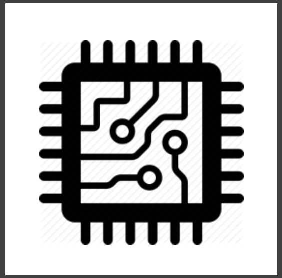
Qubit – a two-state *quantum-mechanical* system

Classical Bit → One out of 2^N possible permutations

A 3-bit register:

Input
100



Output
011

CLASSICAL vs QUANTUM computation

By 2040 we will not have the capability to power all of the machines around the globe (*Semiconductor Industry Association* report).

Industry is focused on finding ways to make computing more energy efficient, but classical computers are limited by the minimum amount of energy it takes them to perform one operation.

This energy limit is named after *Rolf Landauer (IBM Research),* who in 1961 found that in any computer, each single bit operation must use an absolute minimum amount of energy.

$$E_{min} = k_B T \ln 2$$

@ room temperature it is 18 meV or 2.88 $\times 10^{-6}$ fJ

Necessity in turning to radically different ways of computing, such as QUANTUM COMPUTING, to find ways to cut energy use.

Qubit – a two-state *quantum-mechanical* system

Classical Bit → One out of 2^N possible permutations

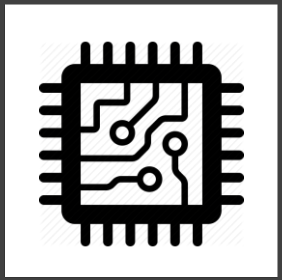
Qubit → All of possible 2^N permutations

Qubits are processed all at the same time!

Exponential speedup

A 3-bit register:

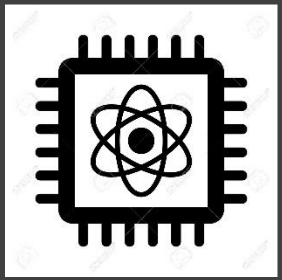
Input
100



Output
011

A 3-Qubit register:

Input
000
001
010
100
110
101
101
111



Output
000
001
010
100
110
101
101
111

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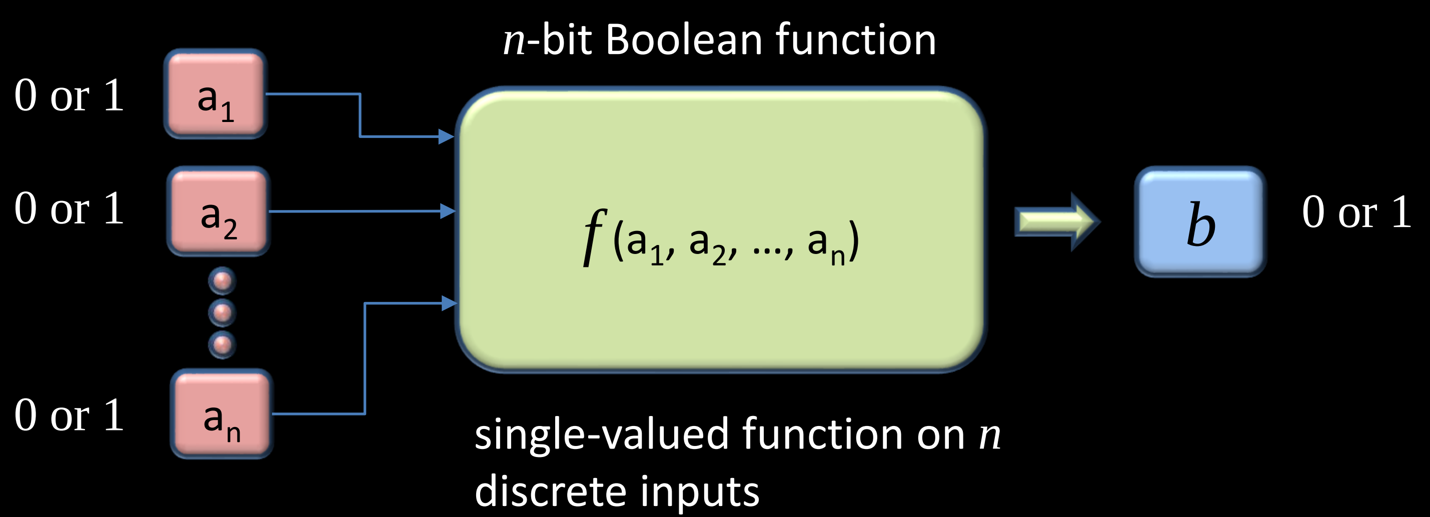
CLASSICAL vs QUANTUM computation



- given an arbitrarily large function *f*, is it possible to identify a universal set of simple functions – called GATEs – that can be used repeatedly in sequence to simulate *f* on its inputs
- each gate is formed by small number of inputs from $a_1, \ldots, a_n$

CLASSICAL vs QUANTUM computation

a_1	a_2	a_1 AND a_2
0	0	0
0	1	0
1	0	0
1	1	1

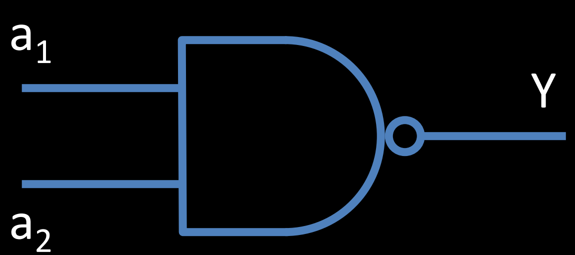
+

a_1	a_2	a_1 OR a_2
0	0	0
0	1	1
1	0	1
1	1	1

+

a_1	NOT a_1
0	1
1	0

simulate arbitrary Boolean functions using the AND, OR, and NOT gates only

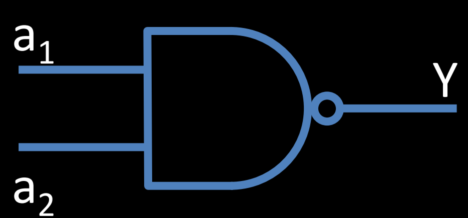


a_1	a_2	a_1 NAND a_2
0	0	1
0	1	1
1	0	1
1	1	0

the number of NAND (2-bit) gates needed to simulate a function with n inputs scales exponentially in n

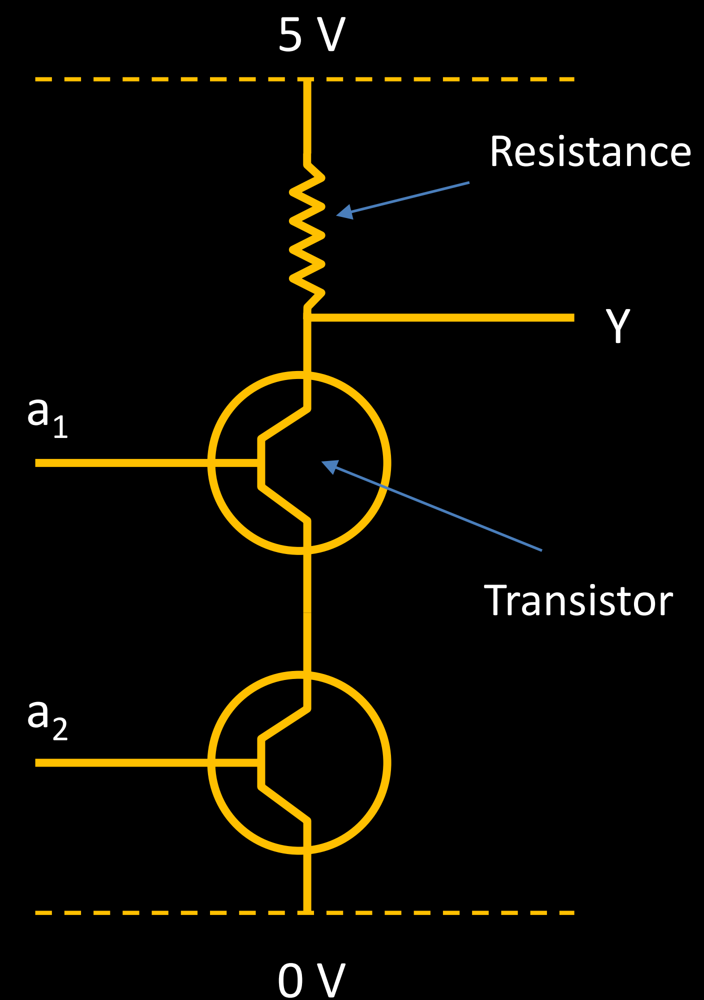
CLASSICAL vs QUANTUM computation

a_1	a_2	a_1 NAND a_2
0	0	1
0	1	1
1	0	1
1	1	0



NAND logic gate

HARDWARE



CLASSICAL vs QUANTUM computation

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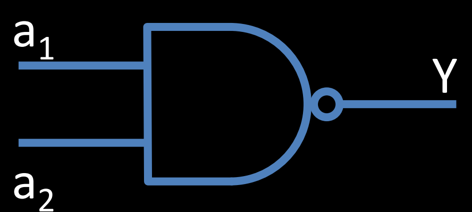
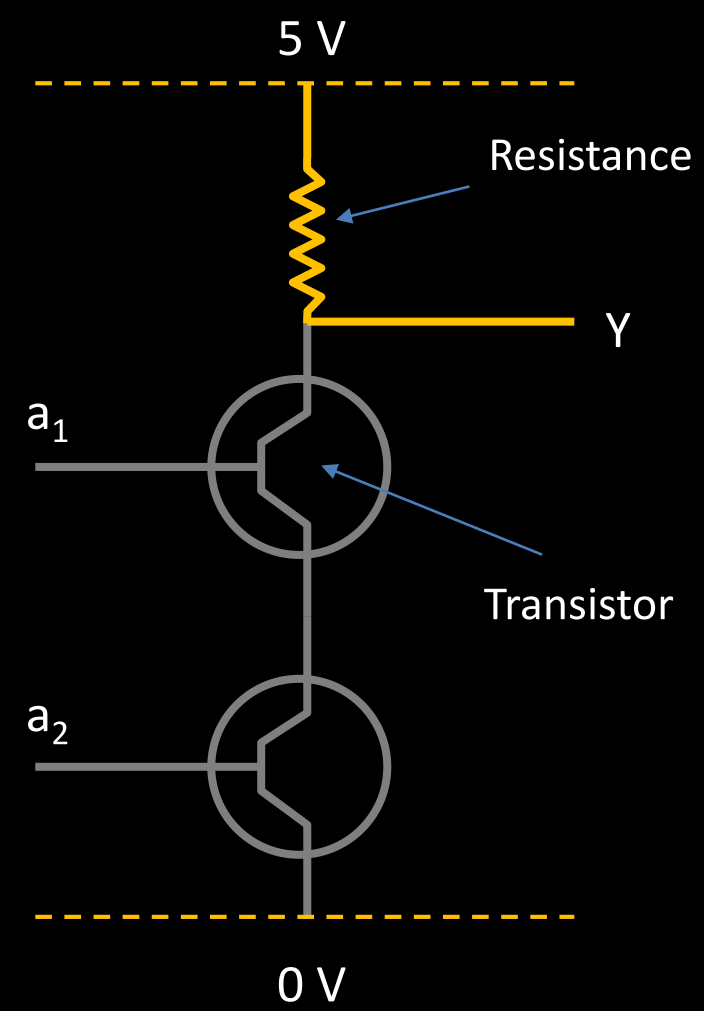


NAND logic gate

HARDWARE



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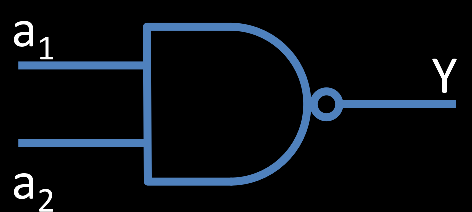
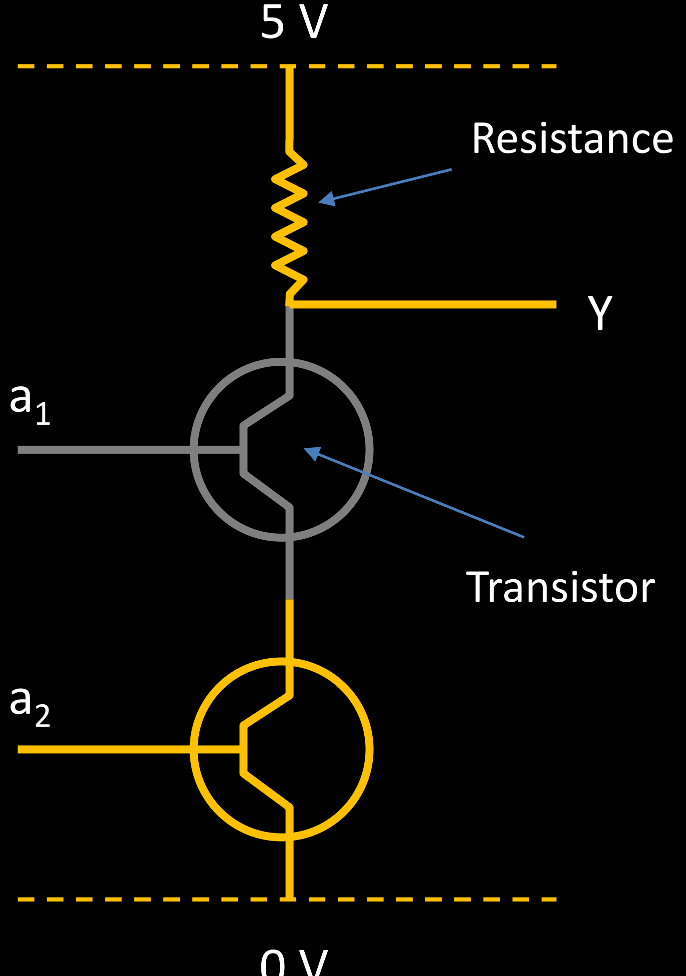
CLASSICAL vs QUANTUM computation

a_1	a_2	a_1 NAND a_2
0	0	1
0	1	1
1	0	1
1	1	0



NAND logic gate

HARDWARE



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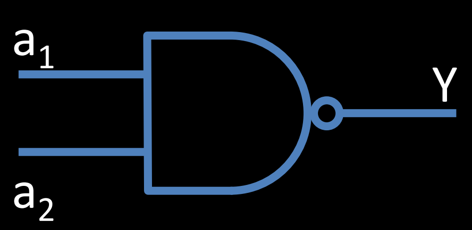
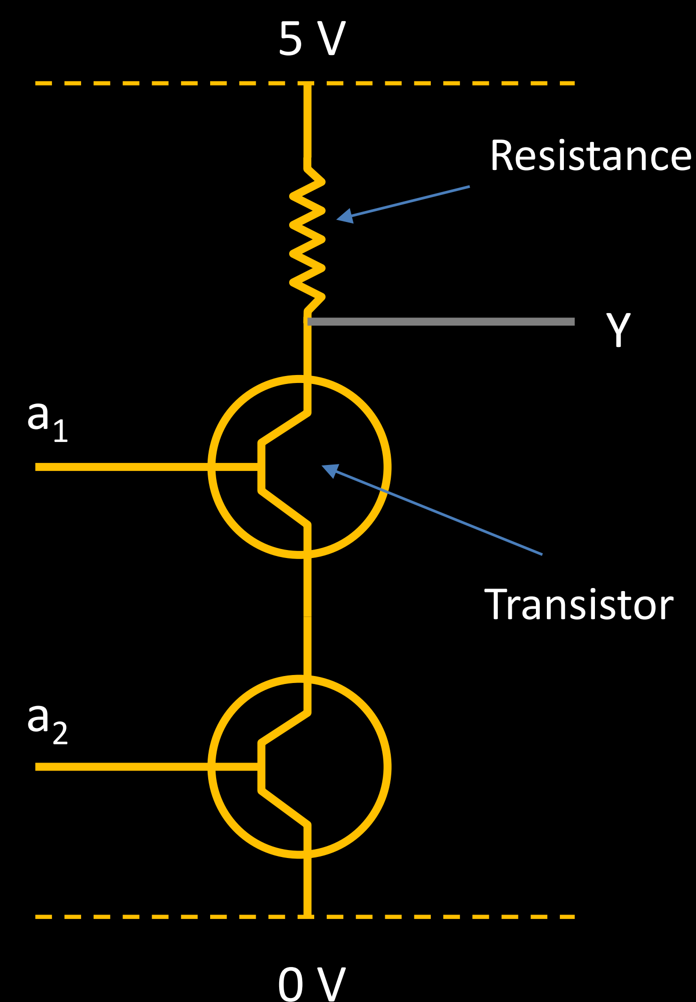
CLASSICAL vs QUANTUM computation

a_1	a_2	a_1 NAND a_2
0	0	1
0	1	1
1	0	1
1	1	0



NAND logic gate

HARDWARE



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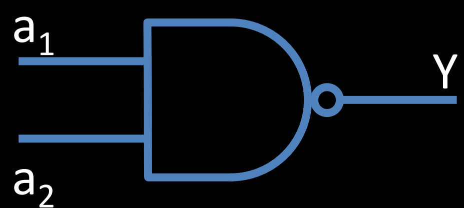
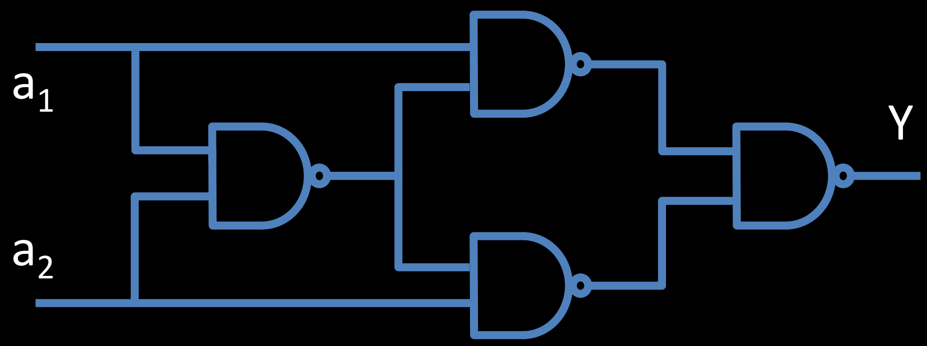
CLASSICAL vs QUANTUM computation

a_1	a_2	a_1 NAND a_2
0	0	1
0	1	1
1	0	1
1	1	0

→ XOR

a_1	a_2	a_1 XOR a_2
0	0	0
0	1	1
1	0	1
1	1	0





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CLASSICAL vs QUANTUM computation

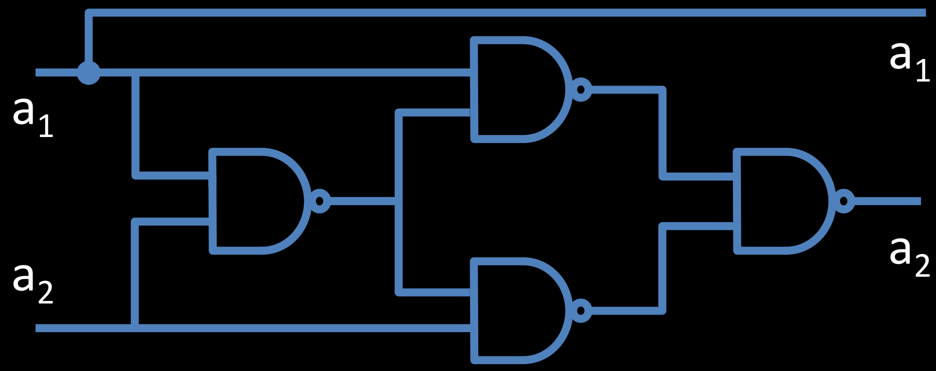
a_1 CONTROL

a_2 TARGET

XOR

a_1	a_2	a_1 XOR a_2
0	0	0
0	1	1
1	0	1
1	1	0

Input		Output	
a_1	a_2	a_1	a_2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



CLASSICAL vs QUANTUM computation

a_1 CONTROL

a_2 TARGET

If the CONTROL bit is set to 0 it does nothing.

If it is set to 1, the TARGET bit is flipped.

That is, the gate causes the target bit to be correlated to the control bit.

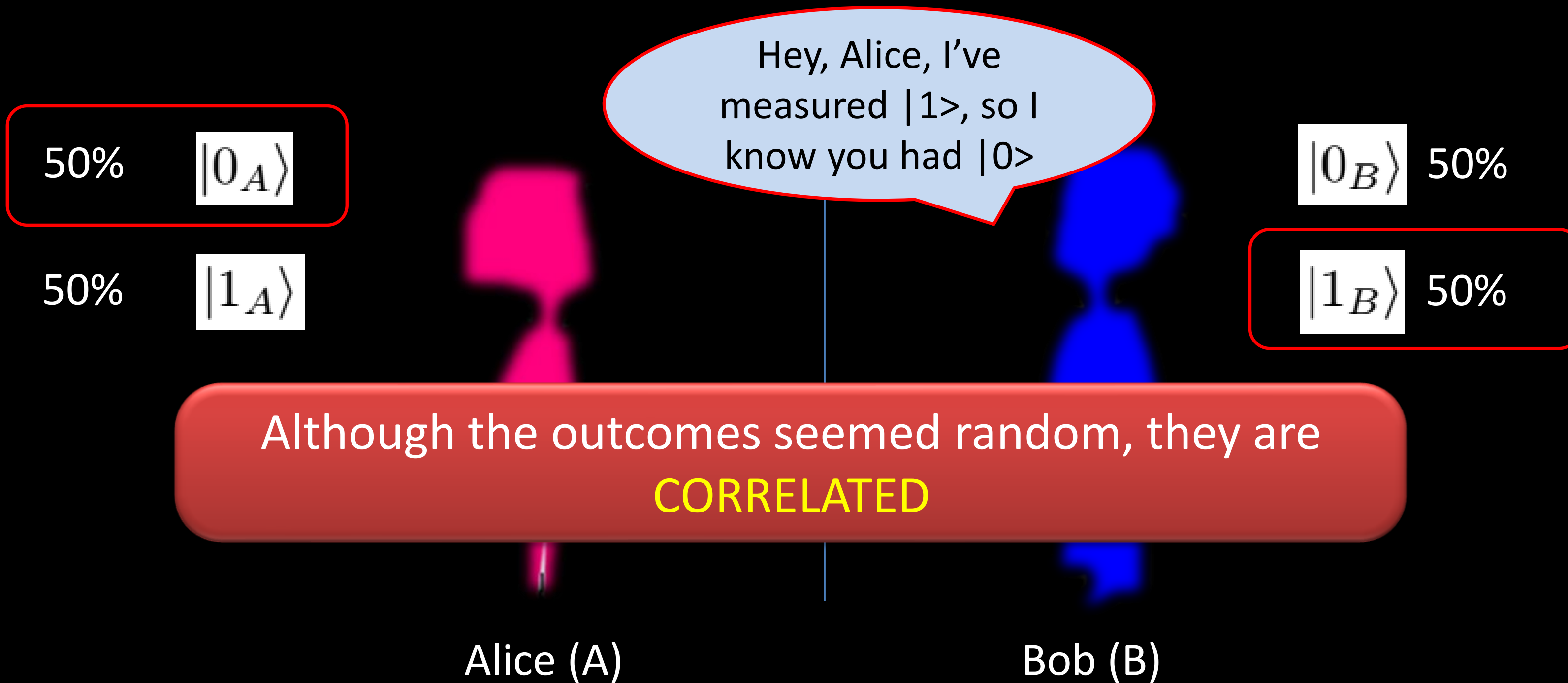
Here comes the “ENTANGLEMENT”

Input		Output	
a_1	a_2	a_1	a_2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Input		Output	
CONTROL	TARGET	CONTROL	TARGET
$\vert 0\rangle$	$\vert 0\rangle$	$\vert 0\rangle$	$\vert 0\rangle$
$\vert 0\rangle$	$\vert 1\rangle$	$\vert 0\rangle$	$\vert 1\rangle$
$\vert 1\rangle$	$\vert 0\rangle$	$\vert 1\rangle$	$\vert 1\rangle$
$\vert 1\rangle$	$\vert 1\rangle$	$\vert 1\rangle$	$\vert 0\rangle$

QUANTUM ENTANGLEMENT

Quantum ENTANGLEMENT is a quantum mechanical phenomenon in which the quantum states of two or more objects have to be described with reference to each other (CORRELATED), even though the individual objects may be spatially separated.



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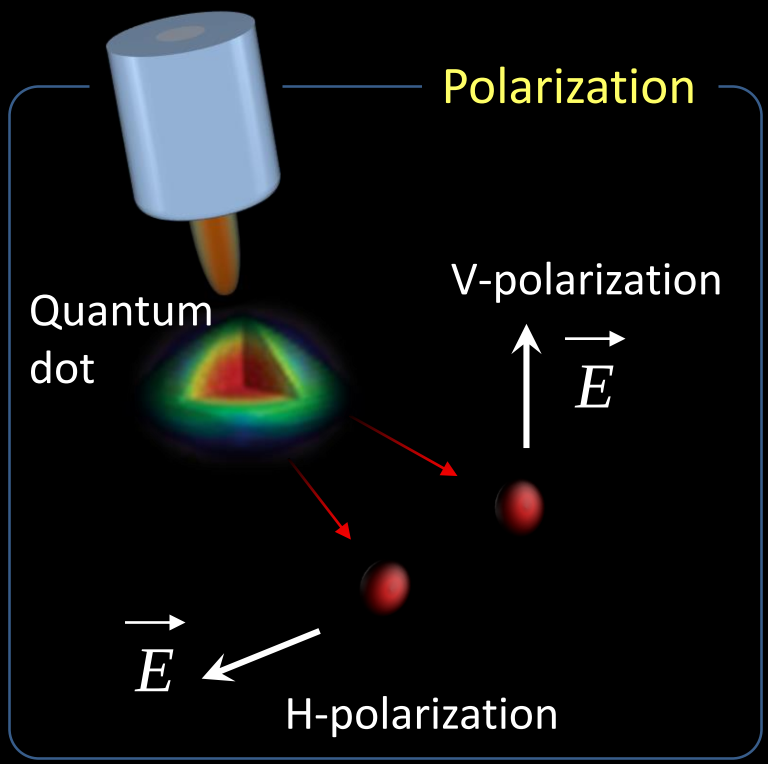

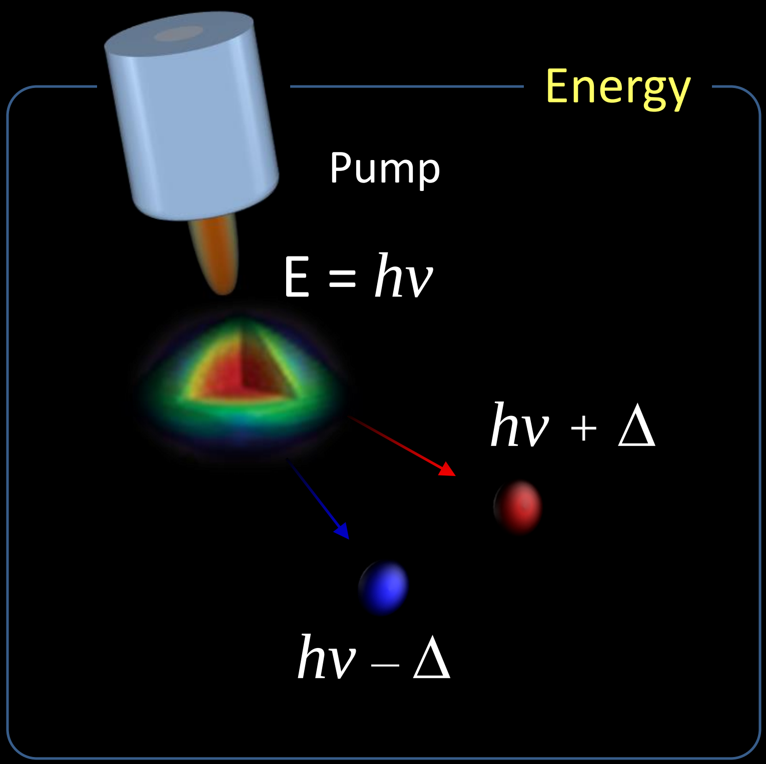

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The quantum C-NOT gate

The CNOT gate is the "quantization" of a classical XOR gate.

It is a quantum gate that is an essential component in the construction of a quantum computer. It can be used to entangle and disentangle quantum states.

Any quantum circuit can be simulated to an arbitrary degree of accuracy using a combination of CNOT gates.

C-NOT (Controlled NOT)

<table>
<tr><th colspan="2">Input</th><th colspan="2">Output</th></tr>
<tr><td>CONTROL</td><td>TARGET</td><td>CONTROL</td><td>TARGET</td></tr>
<tr><td>|0></td><td>|0></td><td>|0></td><td>|0></td></tr>
<tr><td>|0></td><td>|1></td><td>|0></td><td>|1></td></tr>
<tr><td>|1></td><td>|0></td><td>|1></td><td>|1></td></tr>
<tr><td>|1></td><td>|1></td><td>|1></td><td>|0></td></tr>
</table>

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The quantum C-NOT gate

The CNOT gate transforms a 2-qubit state

$$|\psi\rangle = \alpha \times |00\rangle + \beta \times |01\rangle + \gamma \times |10\rangle + \delta \times |11\rangle$$

into

$$|\psi\rangle = \alpha \times |00\rangle + \beta \times |01\rangle + \gamma \times |11\rangle + \delta \times |10\rangle$$

The CNOT gate operates on a quantum register consisting of 2 qubits.

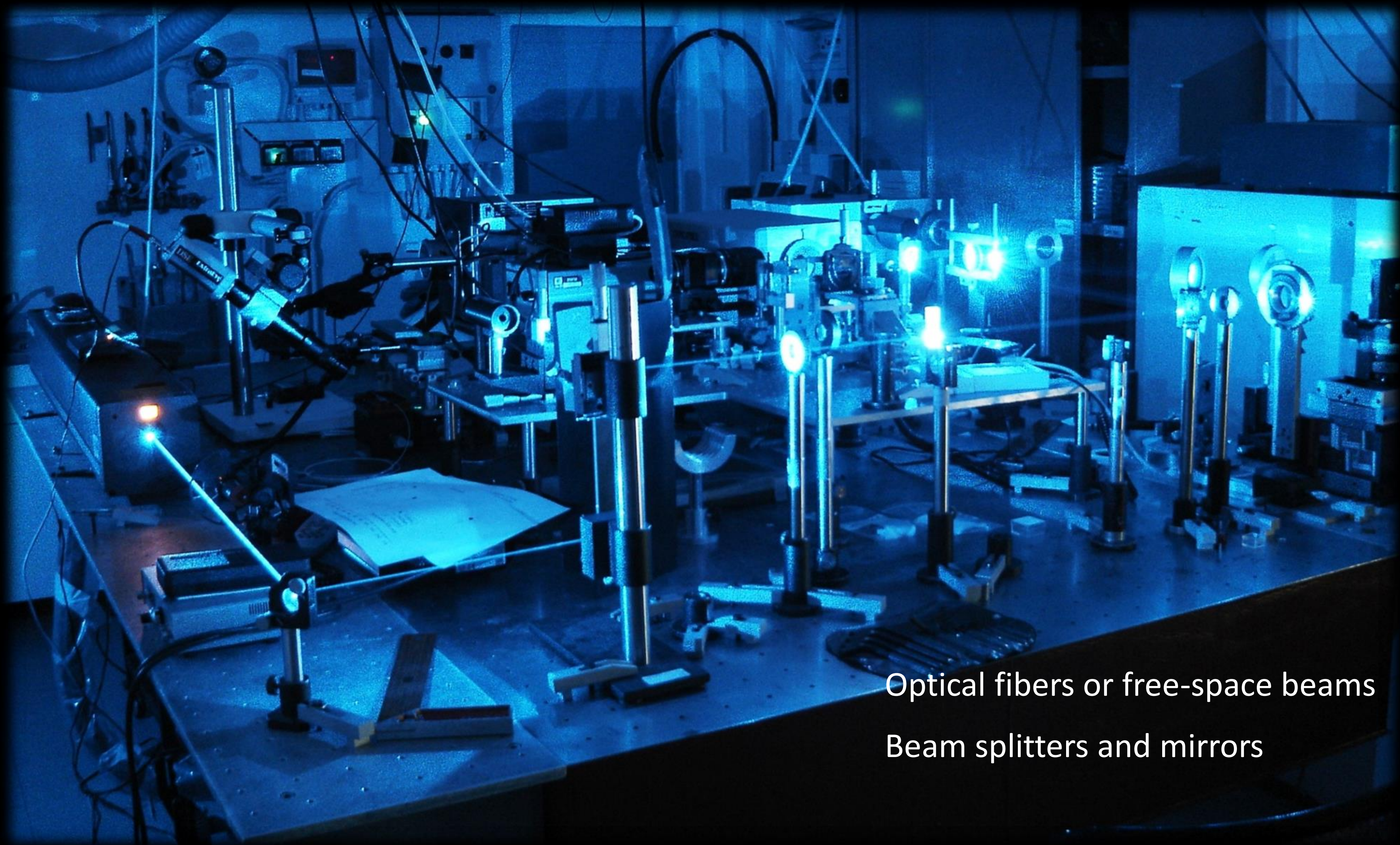
The CNOT gate flips the second qubit (the target qubit) if and only if the first qubit (the control qubit) is $\lvert 1\rangle$

C-NOT (Controlled NOT)

Input		Output	
CONTROL	TARGET	CONTROL	TARGET
$\lvert 0\rangle$	$\lvert 0\rangle$	$\lvert 0\rangle$	$\lvert 0\rangle$
$\lvert 0\rangle$	$\lvert 1\rangle$	$\lvert 0\rangle$	$\lvert 1\rangle$
$\lvert 1\rangle$	$\lvert 0\rangle$	$\lvert 1\rangle$	$\lvert 1\rangle$
$\lvert 1\rangle$	$\lvert 1\rangle$	$\lvert 1\rangle$	$\lvert 0\rangle$

How to realize physically a C-NOT gate?

In an optics lab…



Optical fibers or free-space beams

Beam splitters and mirrors

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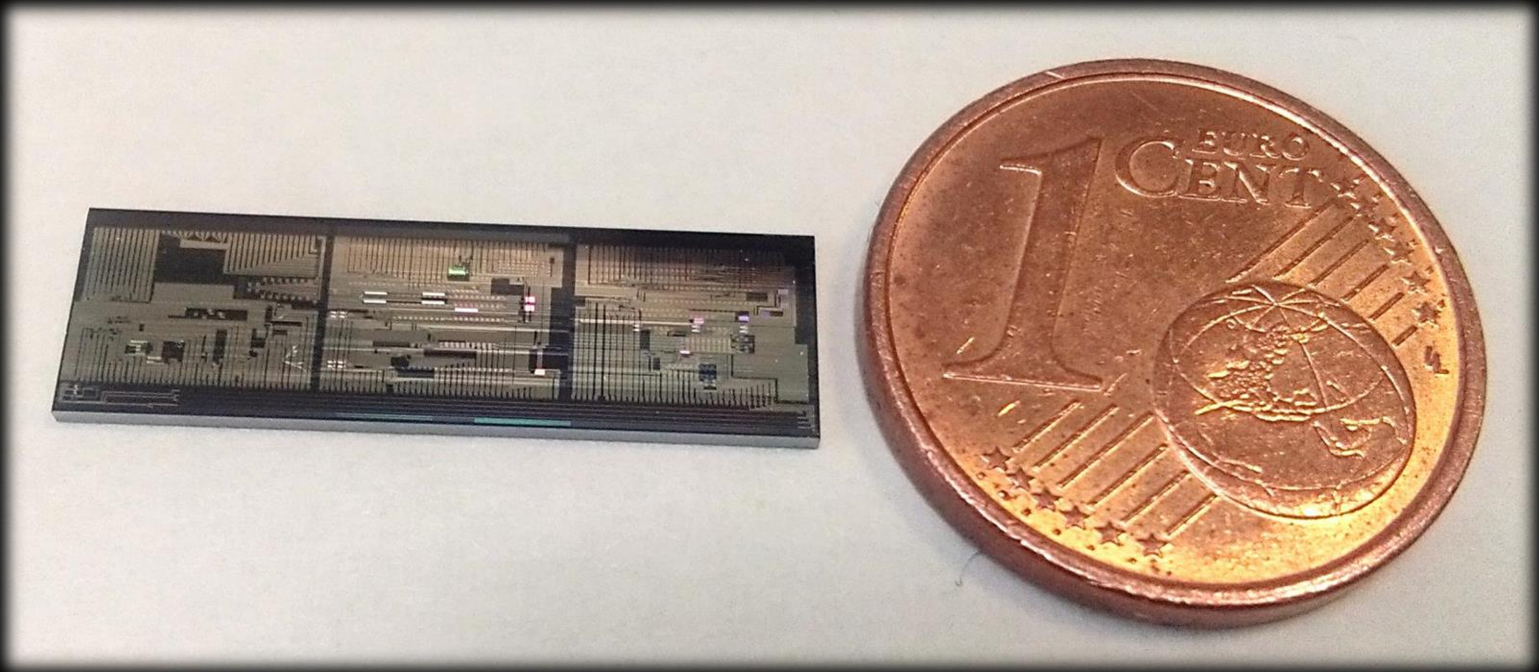
…or integrate these functions into tiny chips

Sqeesing the area by million times !

Volume reduced by 10^{11} times !



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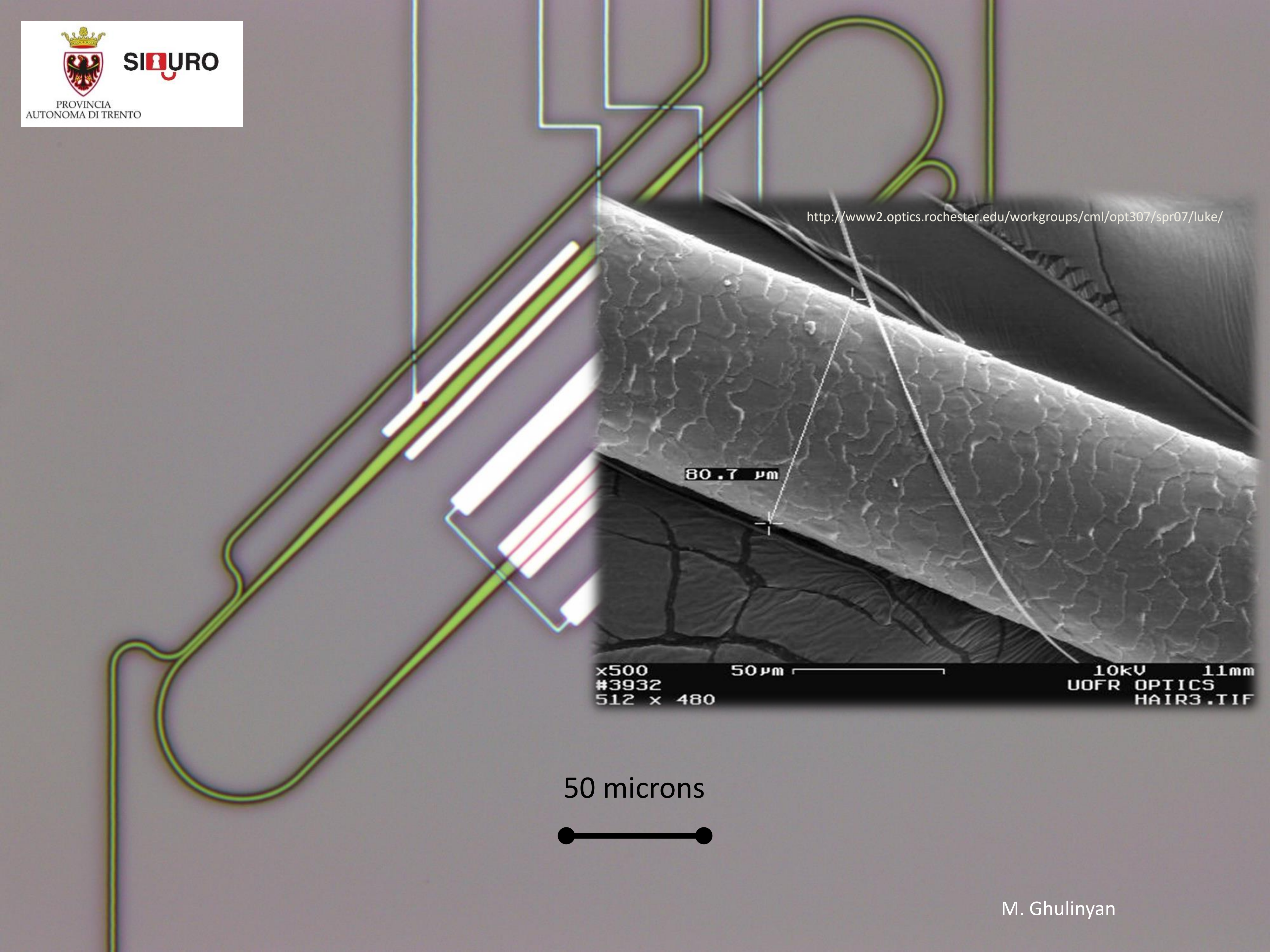
PROVINCIA
AUTONOMA DI TRENTO

SIQURO



50 microns

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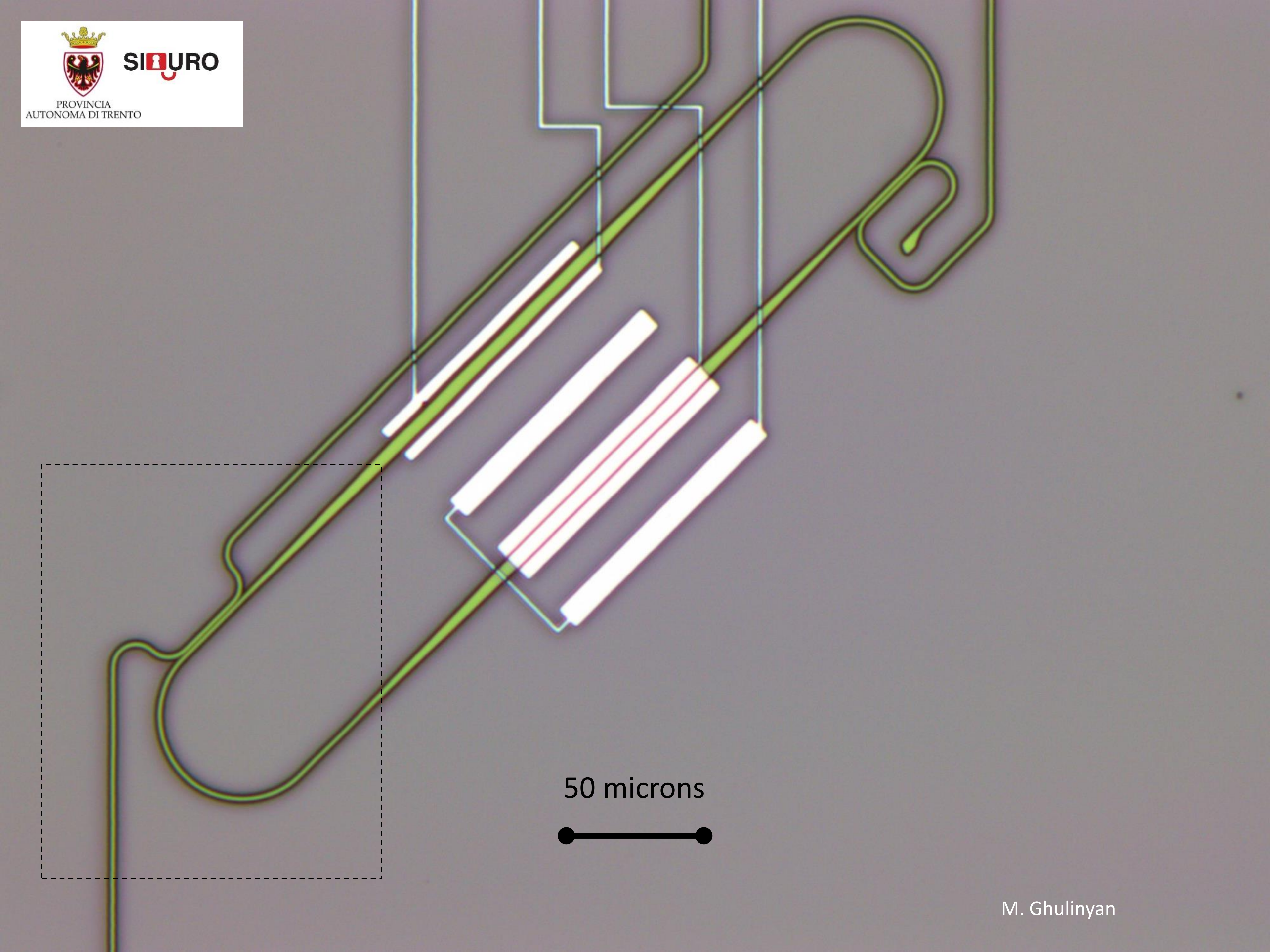
SIQURO
PROVINCIA
AUTONOMA DI TRENTO
50 microns
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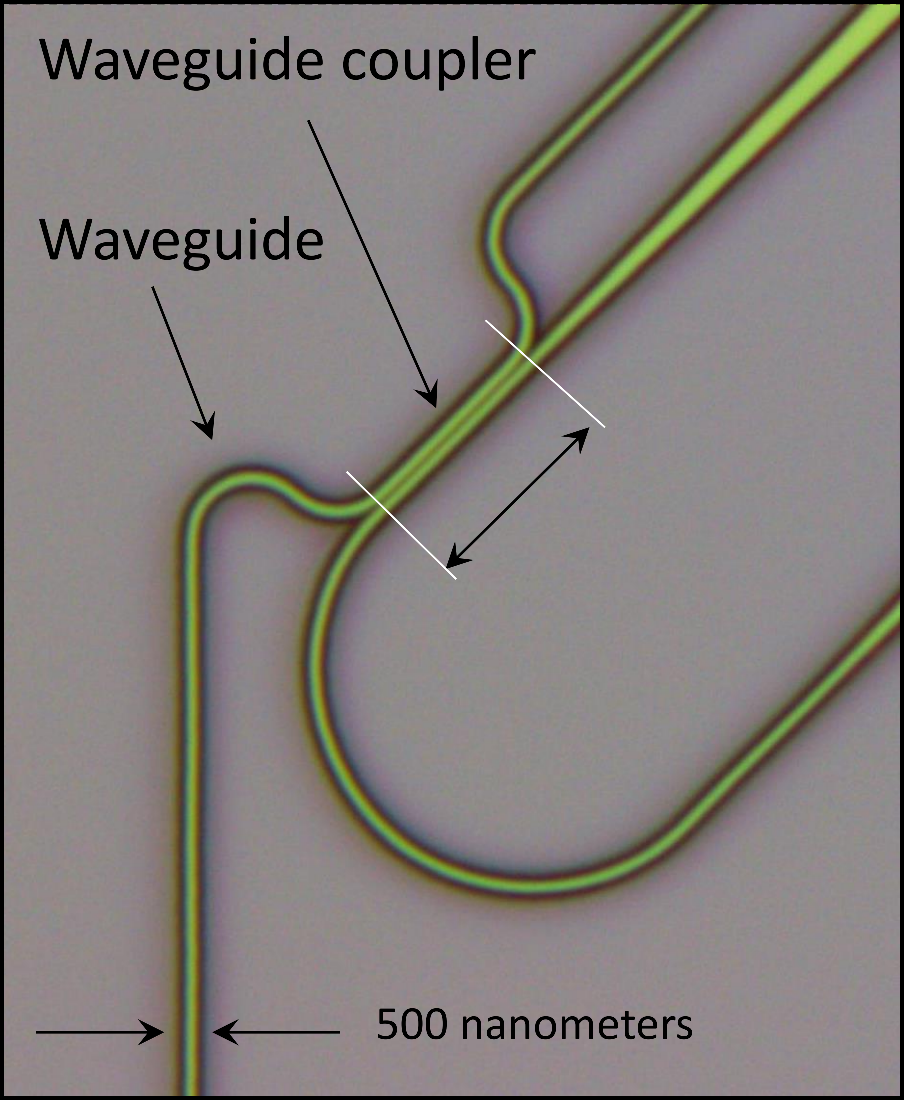

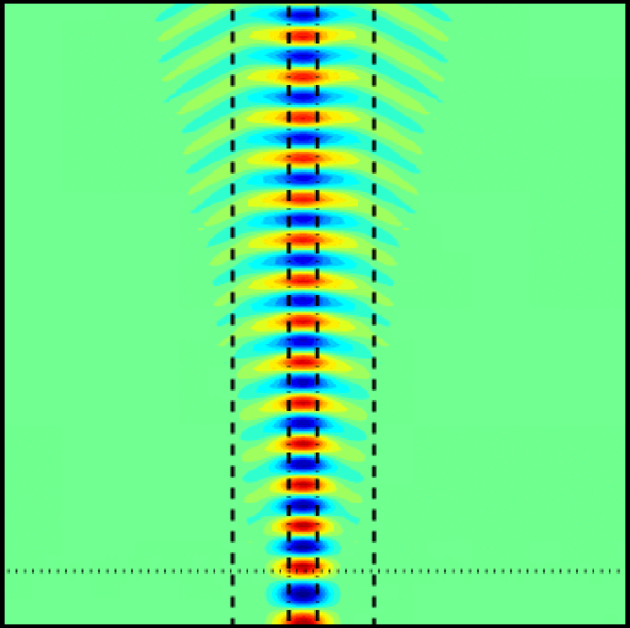

http://www.photonics.umbc.edu/

Waveguide = Optical fiber / Free-beam + mirror

Waveguide coupler = Beam splitter

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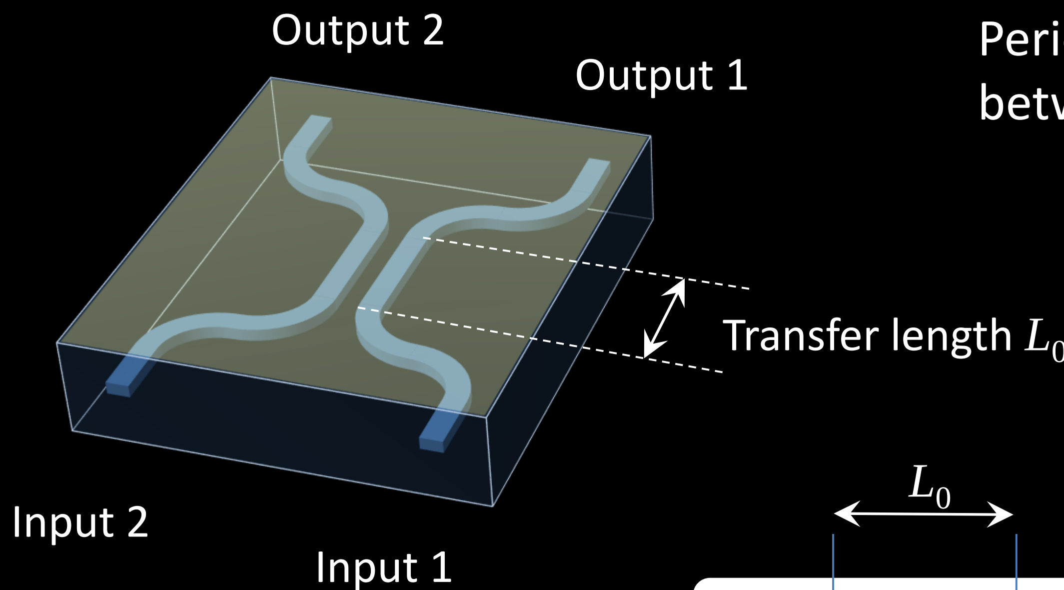
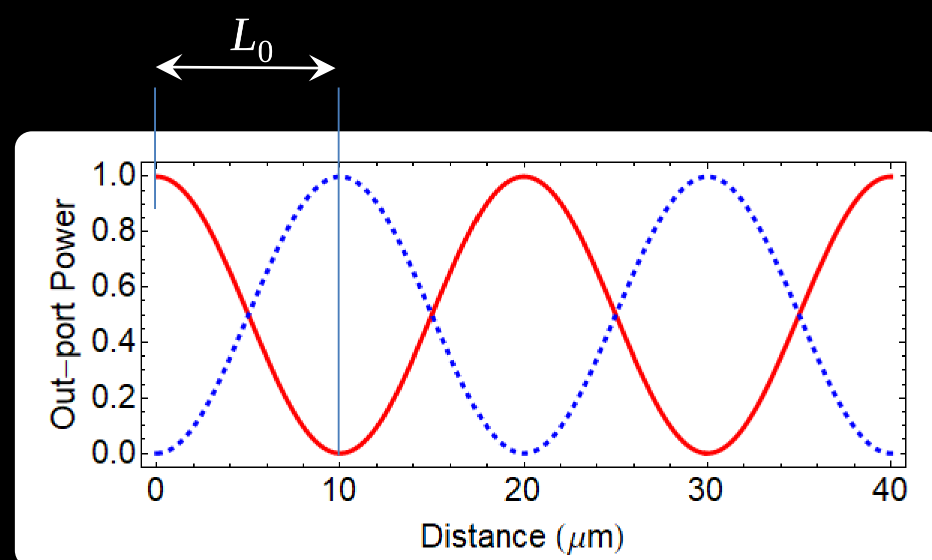
Beam splitter – waveguide coupler



Periodic exchange of power between waveguides



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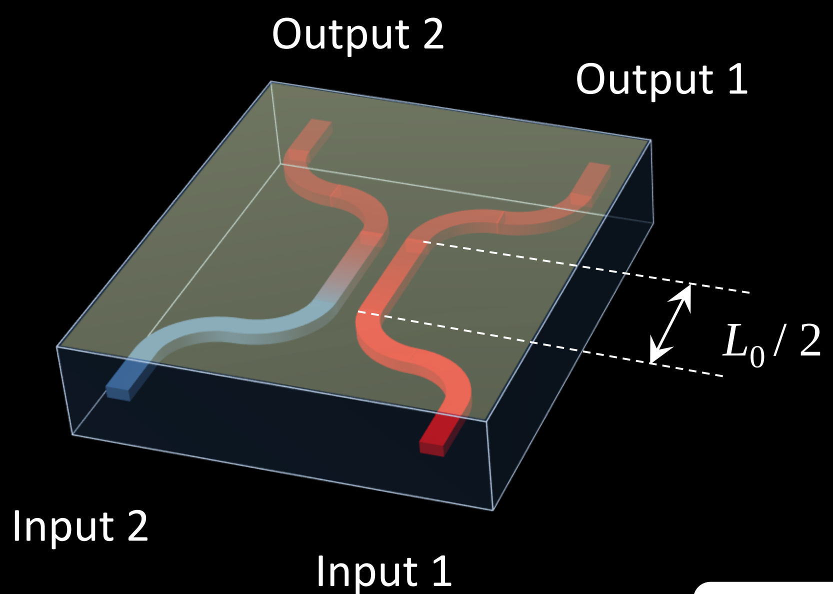
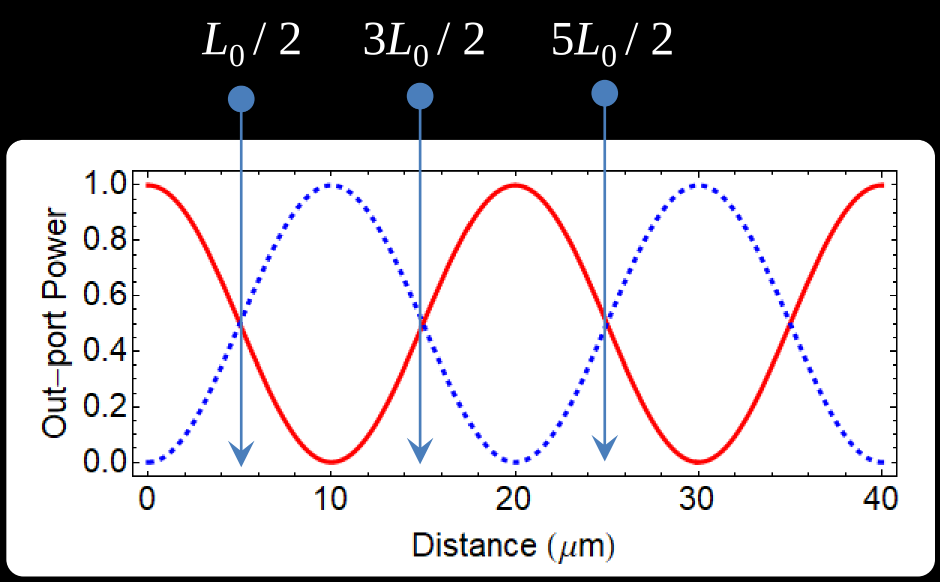
Beam splitter – waveguide coupler



Periodic exchange of power between waveguides



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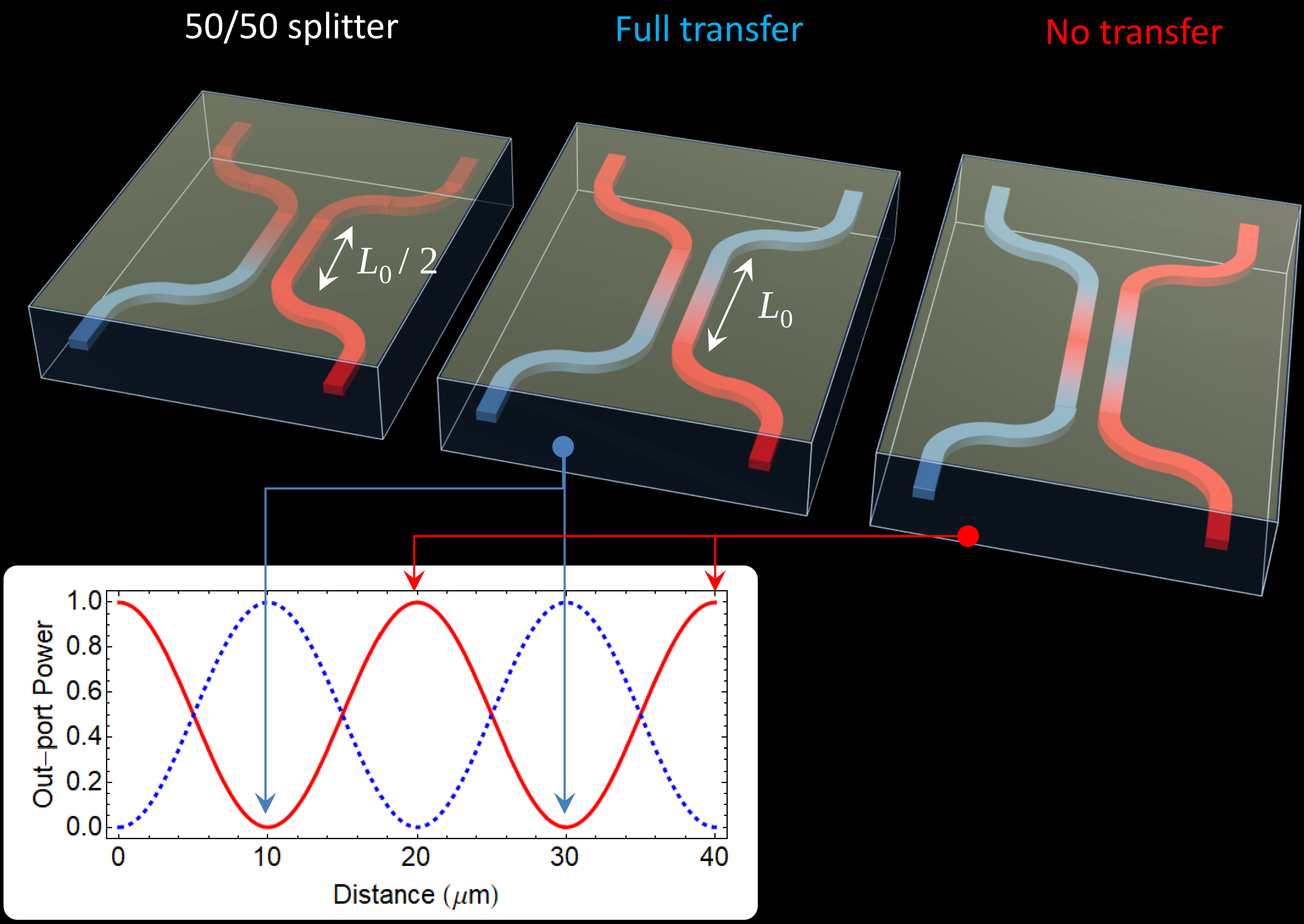
Beam splitter – waveguide coupler



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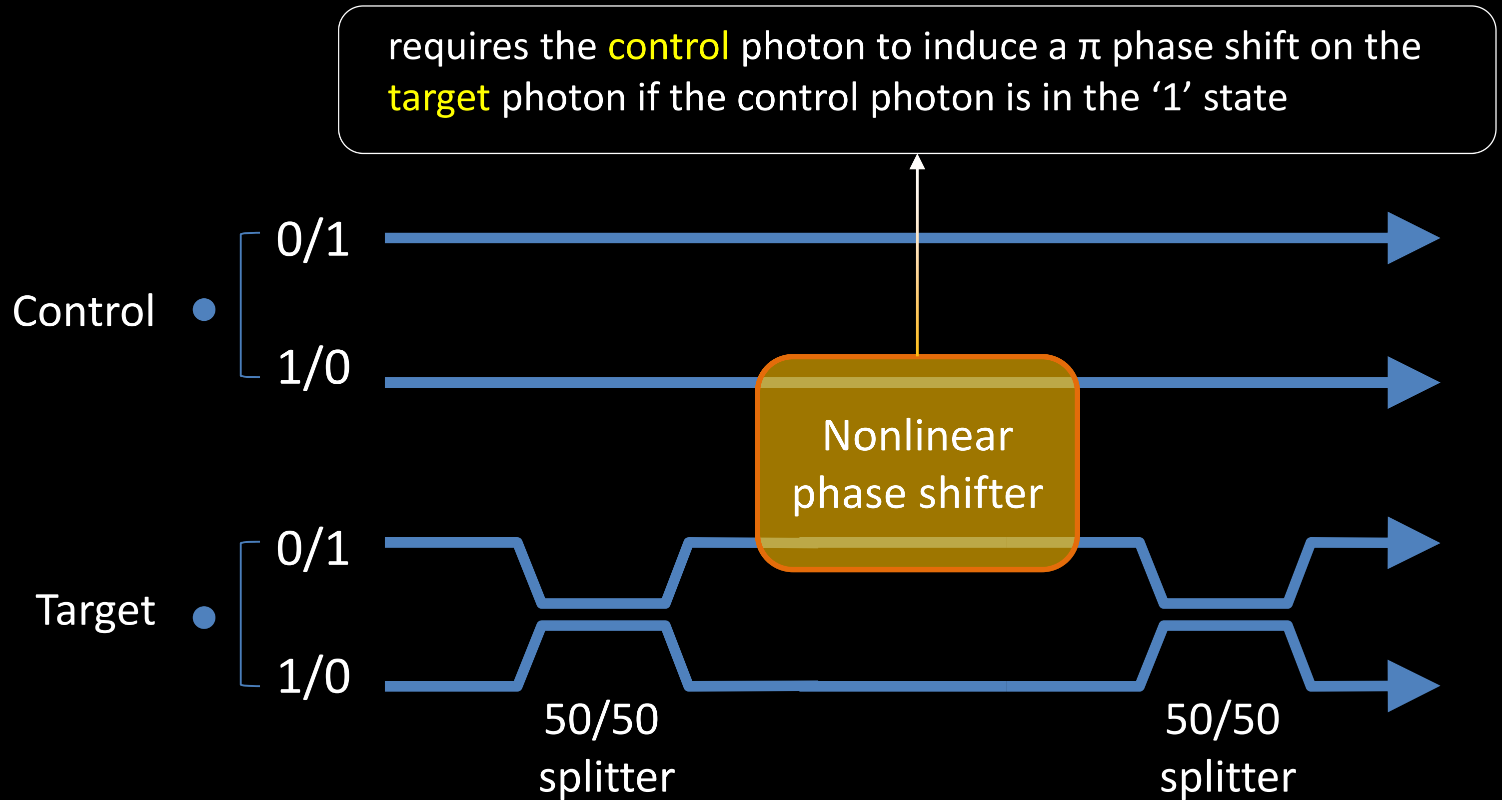
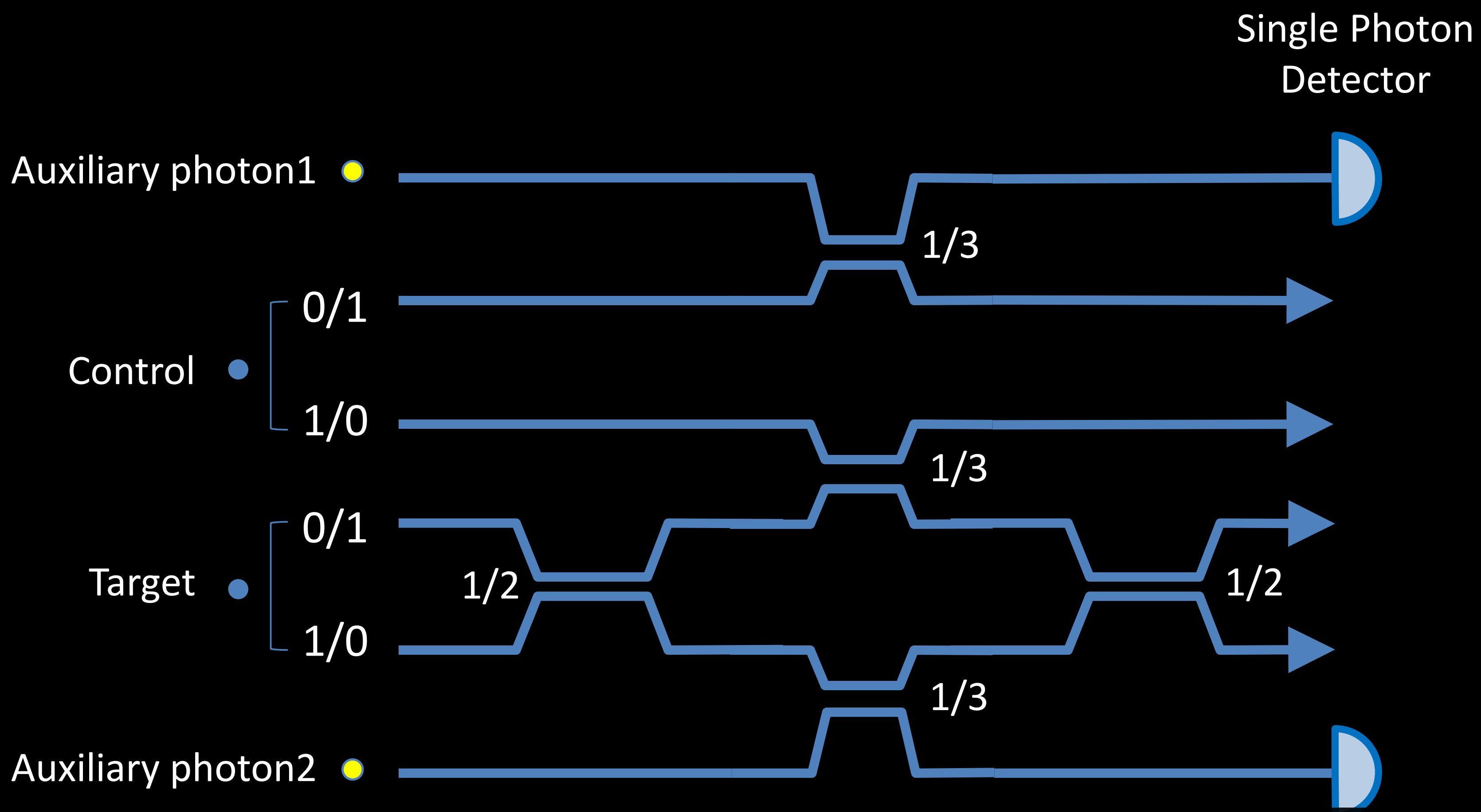
An optical C-NOT quantum gate



O'Brien, Jeremy L., Akira Furusawa, and Jelena Vučković. "Photonic quantum technologies." *Nature Photonics* 3.12 (2009): 687-695.

A linear optical C-NOT quantum gate



The CNOT operation is applied to the control and target qubits, conditional on a single photon being detected at each detector

Ralph, Timothy C., et al. "Linear optical controlled-NOT gate in the coincidence basis." *Physical Review A* 65.6 (2002): 062324.

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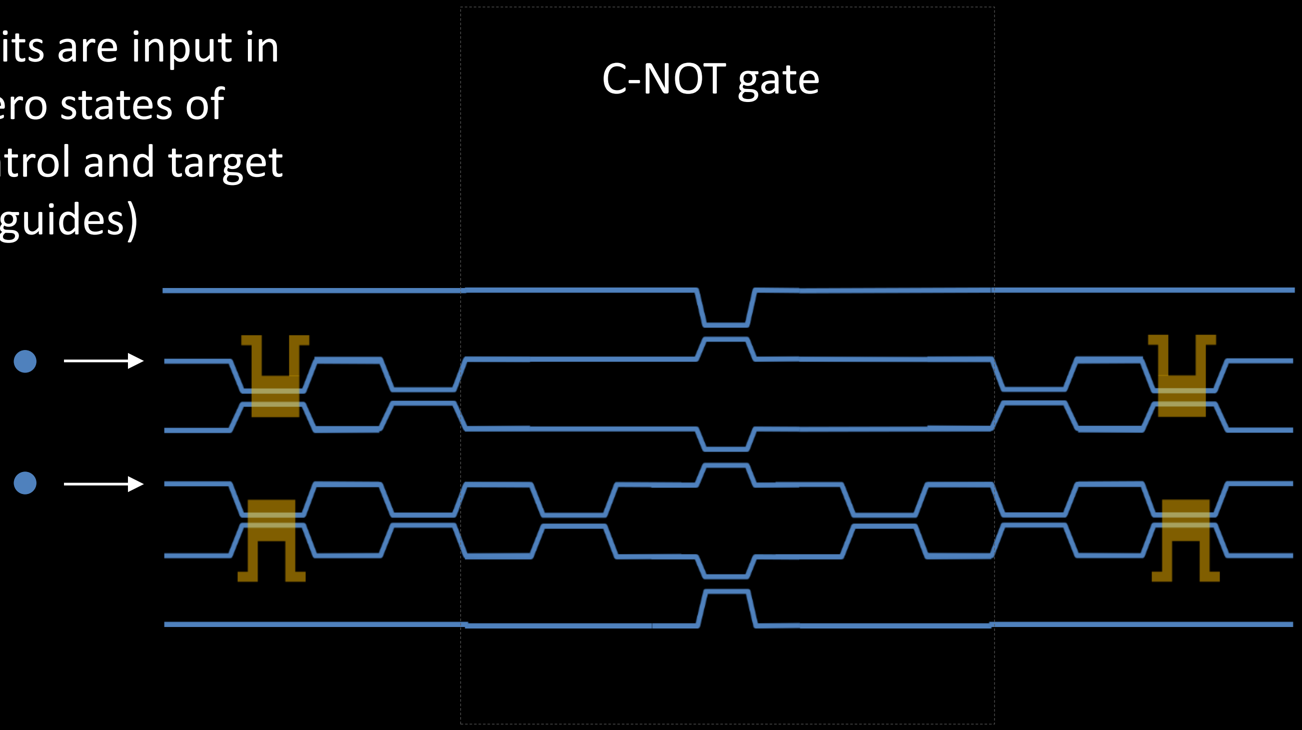
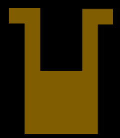
Reconfigurable Quantum circuit

The two qubits are input in the logical zero states of both the control and target (upper waveguides)

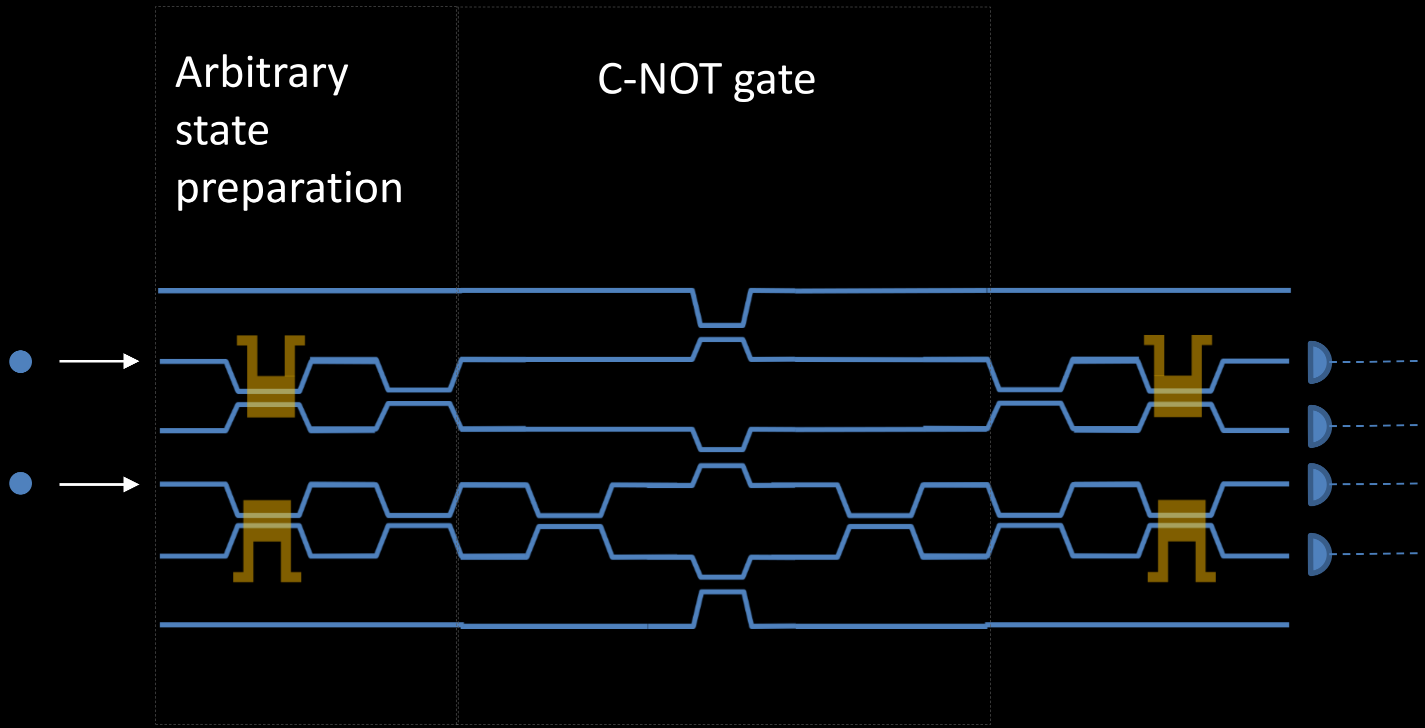


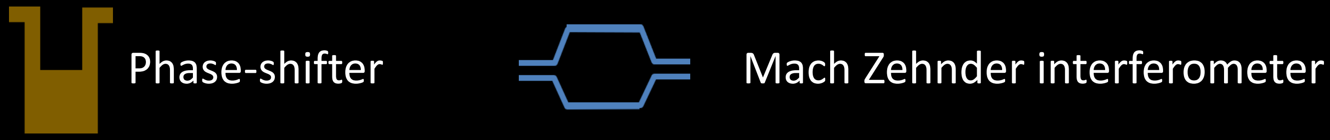

Phase-shifter


Mach Zehnder interferometer

Shadbolt, Peter J., et al. "Generating, manipulating and measuring entanglement and mixture with a reconfigurable photonic circuit." *Nature Photonics* 6.1 (2012): 45-49.

Reconfigurable Quantum circuit





Shadbolt, Peter J., et al. "Generating, manipulating and measuring entanglement and mixture with a reconfigurable photonic circuit." *Nature Photonics* 6.1 (2012): 45-49.

A Quantum simulator

Controllable quantum systems that can be used to mimic other quantum systems.

They have the potential to enable the tackling of problems that are intractable on conventional computers

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

... the physical world is quantum mechanical, and therefore the proper problem is the simulation of quantum physics.

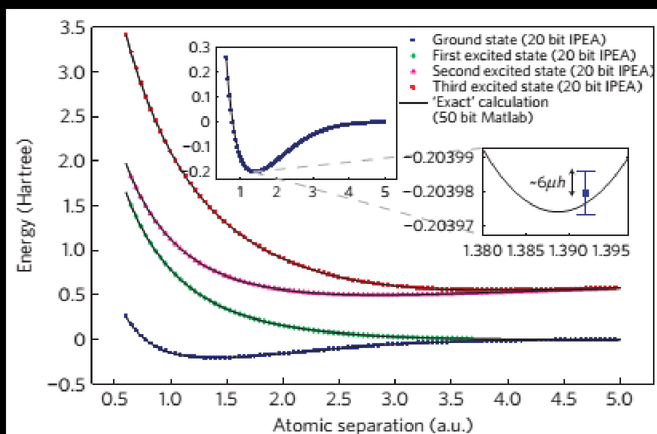
Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws.

...therefore, I believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world.

Quantum simulators

Supercomputers cannot yet predict if a material composed of few hundred atoms will conduct electricity or behave as a magnet, or if a chemical reaction will take place.

Quantum simulators based on the laws of quantum physics will allow us to overcome the shortcomings of supercomputers and to simulate materials or chemical compounds, as well as to solve equations in other areas, like high-energy physics …



Results of quantum optics experiment for simulating the energy of the hydrogen molecule in the minimal basis set. Plot of the molecular energies of the different electronic states as a function of interatomic distance.

Lanyon, B. P. et al. Towards quantum chemistry on a quantum computer. Nature Chem. 2, 106 (2010)

Peng, X., Zhang, J., Du, J. & Suter, D. Quantum simulation of a system with competing two- and three-body interactions. *Phys. Rev. Lett.* **103,** 140501 (2009).

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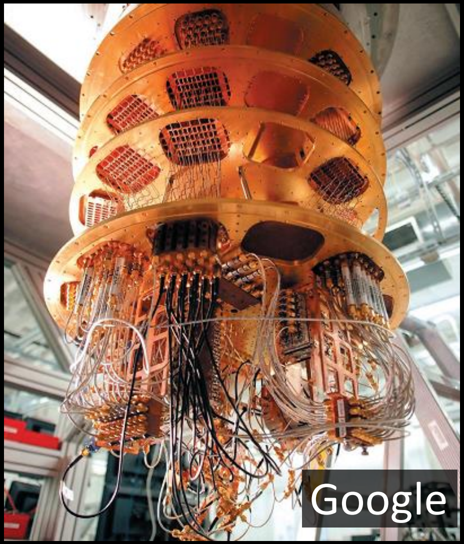
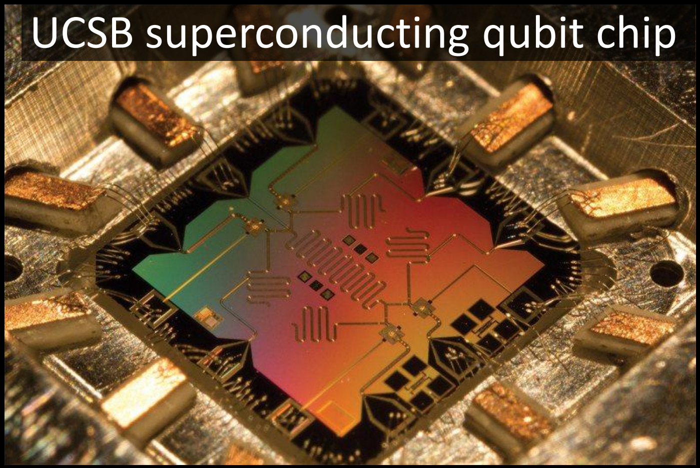
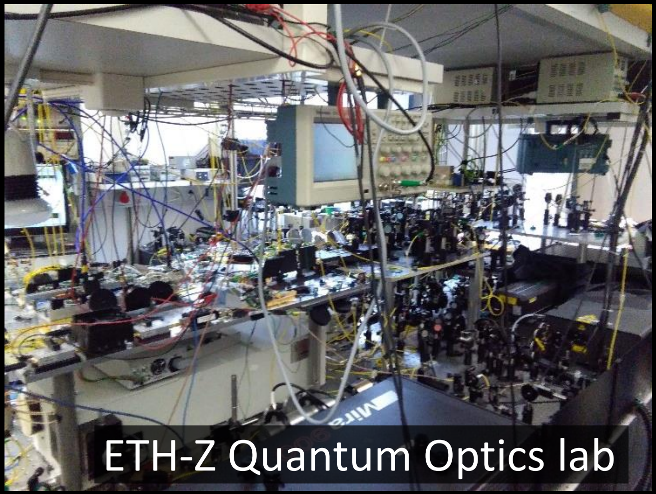
Bulky quantum simulators/computers







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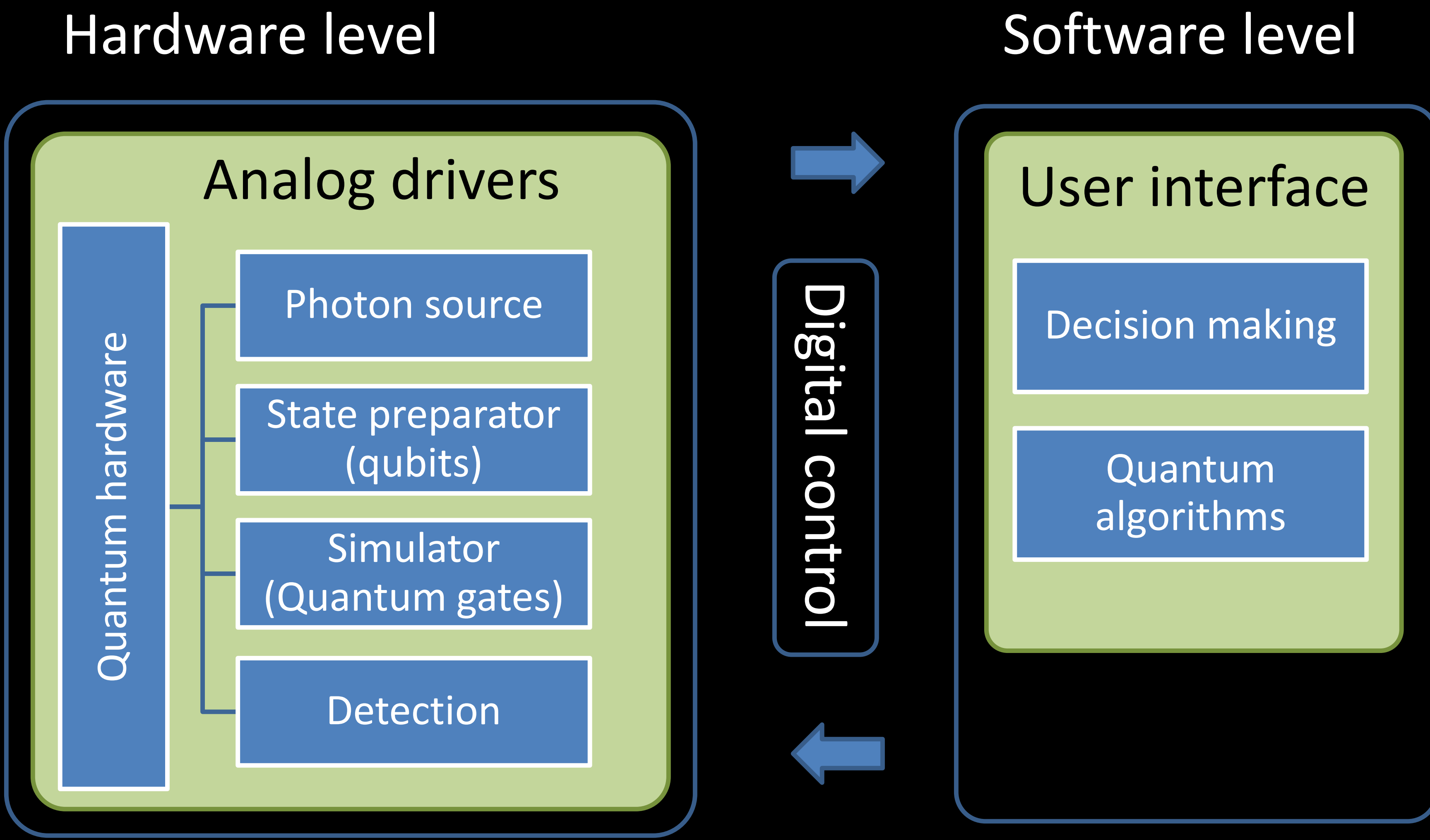
Photonic Quantum simulator: ingredients



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INQUEST - Hybrid integrated photonic-electronic platform for quantum simulations

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INQUEST – the concept

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INQUEST – our vision

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INQUEST – on chip photon source

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Quantum computers are about to get real

As the first qubit-based machines come online, scientists imagine the possibilities

BY EMILY CONOVER

SEEKING QUANTUM SUPREMACY Google, IBM and others are developing quantum computers. IBM has made its five-qubit computer accessible online for free, and the company is planning a 50-qubit computer for commercial use. The large tank shown here holds the cooling system that keeps a 17-qubit processor cold.

Magazine issue: Vol. 191, No. 13, July 8, 2017, p. 28

nature INSIGHT

QUANTUM SOFTWARE

14 September 2017 / Vol 549 / Issue No 7671

Decades have passed since the great minds of physics, including Richard Feynman and David Deutsch, predicted that the laws of quantum mechanics could give rise to a computing paradigm that — for certain tasks — is superior to classical computing. But controlling fragile quantum systems well enough to construct even the most primitive quantum computing hardware has proved taxing.

Experimental advances in the past few years have hushed the sceptics of quantum computing. However, the point that it is not entirely clear which application of quantum computers will redeem the hard work remains valid. This Insight discusses the applications in which quantum computers may excel and how software will actually run on such machines.

Just as programming languages and compilers facilitate interaction with the semiconductor transistors in a classical computer, many layers of software tools will sit between quantum algorithms and hardware. An important component is quantum error-correcting

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172 Roads towards fault-tolerant universal quantum computation
Earl T. Campbell, Barbara M. Terhal & Christophe Vuillot

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Quantum Software[1]

The advent of fully fledged, universal quantum computers will signify a radical departure from current computing. Several aspects include:

- Quantum programming languages and compilers
- fault-tolerant quantum computation
- quantum algorithms
- post-quantum cryptography

[1] Leonie Muec. "Quantum software". In: *Nature Insight* 549.7671 (2017), 171—209.

Quantum computing and Machine Learning[2]

- Machine learning $\longrightarrow$ quantum systems

- Quantum Systems $\longrightarrow$ Machine Learning

[2] Jacob Biamonte et al. "Quantum machine learning". In: *Nature* 549.7671 (2017). arXiv: 1611.09347.

Quantum computing and Machine Learning[2]

- Machine learning $\longrightarrow$ quantum systems
  - use machine learning techniques for analysing and simulating the behavious of phisical quantum devices
  - machine learning to tackle noise, tailor gates and develop core quantum information processing building blocks.
- Quantum Systems $\longrightarrow$ Machine Learning

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Quantum computing and Machine Learning[2]

- Machine learning $\longrightarrow$ quantum systems
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- Quantum Systems $\longrightarrow$ Machine Learning
  - develop and tailor these quantum methods to apply to problems when facing **big data sets**
  - **Adiabatic quantum optimization**. Adiabatic quantum computing relies on the idea of embedding a problem instance into a physical system, such that the system's lowest energy
  - **Gibbs Sampling:** developing a Gibbs state preparation and sampling protocol, also with the objective of training deep belief networks.

[2] Jacob Biamonte et al. "Quantum machine learning". In: *Nature* 549.7671 (2017). arXiv: 1611.09347.

Qubits

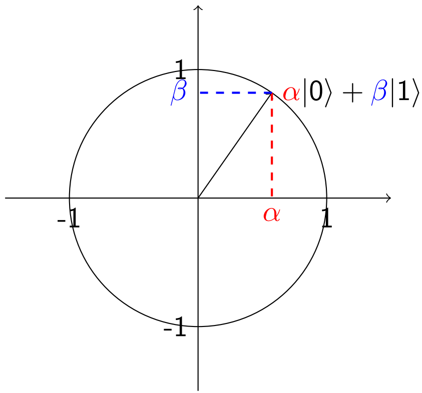
- Quantum mechanics tells us that any such system can exist in a superposition of states.
- In general, the state of a quantum bit (or qubit for short) is described by:

$$\alpha|0\rangle + \beta|1\rangle$$

where, α and β are complex numbers, satisfying

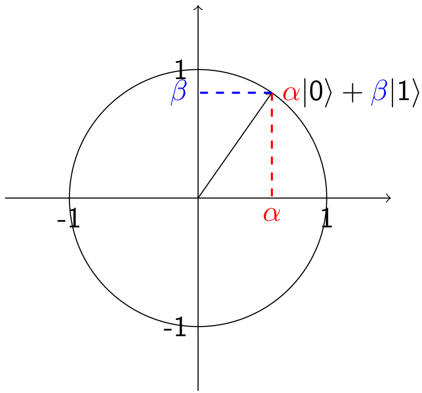
$$|\alpha^2| + |\beta^2| = 1$$

Single qbit - intuitive visualization (when α and β are real numbers)



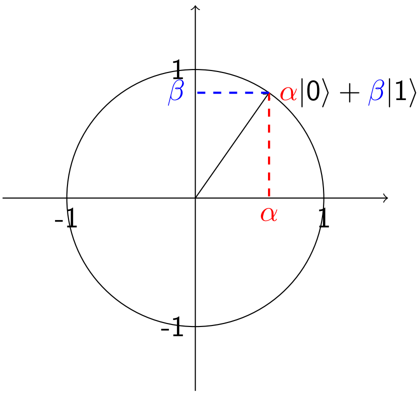
- A qubit may be visualised as a unit vector on the plane.

Single qbit - intuitive visualization (when α and β are real numbers)



- A qubit may be visualised as a unit vector on the plane.
- α^2 and β^2, can be seen as a probability distribution of the two "classical states" $|0\rangle$ and $|1\rangle$ (since $\alpha^2 + \beta^2 = 1$ and $\alpha^2, \beta^2 \geq 0$;

Single qbit - intuitive visualization (when α and β are real numbers)



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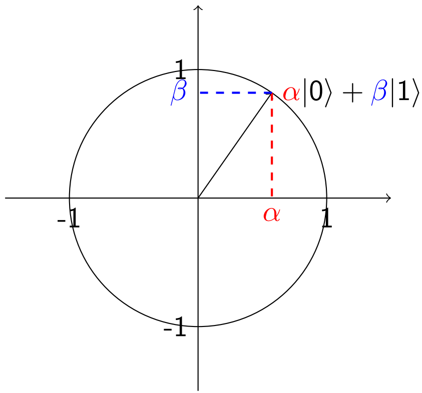

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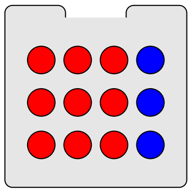
Measuring a qbit

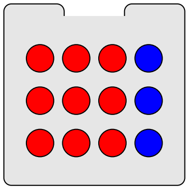
- qbit states are not directly observable;



$$\sqrt{\frac{3}{4}}\,|0\rangle + \sqrt{\frac{1}{4}}\,|1\rangle$$

Measuring a qbit

- qbit states are not directly observable;
- The measuring of a qbit is a stochastic process;
- In measuring $\alpha\,|0\rangle + \beta\,|1\rangle$, we obtain:
  - $|0\rangle$ with probability $|\alpha|^2$
  - $|1\rangle$ with probability $|\beta|^2$



measuring the qbit is like random picking a ball from a urne containing red and blue balls;

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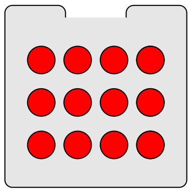
  - $|0\rangle$ with probability $|\alpha|^2$
  - $|1\rangle$ with probability $|\beta|^2$
- after the measure the qbit will go in the observed "classical" state.



$1\,|0\rangle + 0\,|1\rangle$

measuring the qbit is like random picking a ball from a urne containing red and blue balls; whith the side effects the all the balls becomes of the same color of the picked ball

Multiple Qbits super-positions

The superpositions of n qbits, is not just the product of the superpositions of the single qbits. i.e., a set of qbits cannot be seen as a tuple of single q-bits

$$\overbrace{\alpha_1|0\rangle + \beta_1|1\rangle}^{qbit\ 1}, \ldots, \overbrace{\alpha_n|0\rangle + \beta_n|1\rangle}^{qbit\ n}$$

but it is rather the entire tuple of qbits that can be in a super position i.e.,

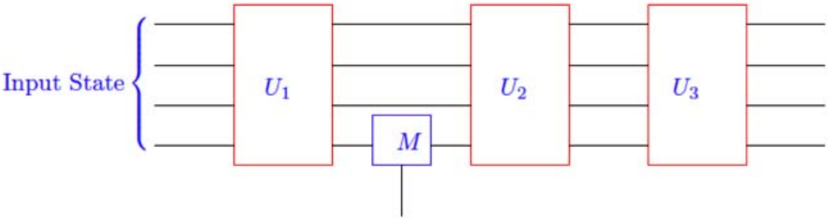
$$\alpha_1|0\ldots00\rangle + \alpha_2|0\ldots01\rangle + \alpha_3|0\ldots10\rangle + \alpha_4|0\ldots11\rangle + \ldots\alpha_{2^n}|1\ldots11\rangle$$

with

$$\sum_{i=1}^{2^n}|\alpha_i^2| = 1$$

Quantum Computation

A quantum circuit is a sequence of unitary operations and measurements on an n-qubit state.



each $\mathcal{U}_i$ is described by a $2^n \times 2^n$ matrix.

Quantum evolutions

- Causality Principle: the evolution of a quantum sistem is described by some function $\mathcal{U}_{\sqcup}$ that depends on time;

$$|\phi(t)\rangle = \mathcal{U}_t \, |\phi(0)\rangle$$

- each $\mathcal{U}_t$ produces a legal quantum state (i,e, $|| \mathcal{U}_t \, |\phi\rangle || = 1$);
- each $\mathcal{U}_t$ is a linear transformation;

Quantum gates - (examples)

Not Gate	$\|0\rangle \overset{\neg}{\mapsto} \|1\rangle$	$\|1\rangle \overset{\neg}{\mapsto} \|0\rangle$
Phase Flip Gate	$\|0\rangle \overset{F}{\mapsto} \|0\rangle$	$\|1\rangle \overset{F}{\mapsto} -\|1\rangle$
Hadamard Gate	$\|0\rangle \overset{H}{\mapsto} \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	$\|1\rangle \overset{H}{\mapsto} \frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$

The search problem

- We want to search for some good item in an unordered N-element search space
- Model this as function

$$f : \{0,1\}^n \to \{0,1\} \qquad \text{with } N = 2^n;$$

$$f(x) = \begin{cases} 1 & \text{If } x \text{ is a solution} \\ 0 & \text{Otherwise} \end{cases}$$

- Classically this takes $O(N)$ steps (queries to f)
- Grover's algorithm does it in $O(\sqrt{N})$ steps

The Grover algorithm for search

- Define the following boolean gate:

$$O_f(x) := (-1)^{f(x)} |x\rangle$$

- suppose that $f(x,y) = \neg(\neg x \vee y)$, then

$$O_f = \begin{array}{ccc} |00\rangle & \rightarrow & |00\rangle \\ |01\rangle & \rightarrow & |01\rangle \\ |10\rangle & \rightarrow & -|10\rangle \\ |11\rangle & \rightarrow & |11\rangle \end{array}$$

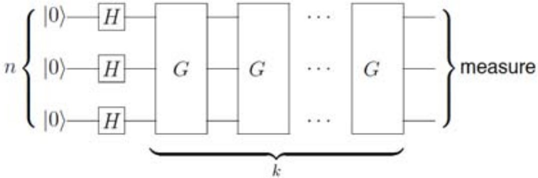
- Let see how O_f transform a qbit in superpositions:

$$O_f(\ \alpha_1 |00\rangle \ +\alpha_2 |01\rangle \ +\alpha_3 |10\rangle \ +\alpha_4 |11\rangle\ )$$

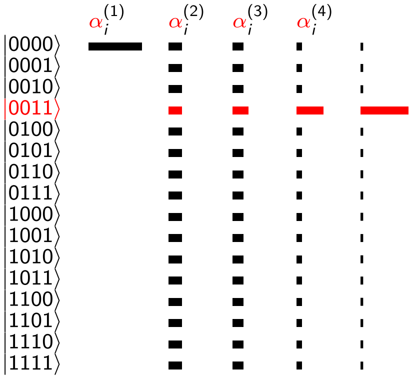
$$=$$

$$\alpha_1 |00\rangle \ +\alpha_2 |01\rangle \ -\alpha_3 |10\rangle \ +\alpha_4 |11\rangle$$

Grover Algorithm



$$G = H^{\otimes n} R H^{\otimes n} O_f$$



What Quantum Computing can do for AI

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